Traffic behavior near an off ramp in the cellular automaton traffic model

Bin Jia,[1,2] Rui Jiang,[2] and Qing-Song Wu[2,*]
[1]*Institute of Systems Science, School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, People's Republic of China*
[2]*School of Engineering Science, University of Science and Technology of China, Hefei 230026, People's Republic of China*
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In this paper, we investigate the off-ramp system on the highway using the cellular automaton traffic model. Both the system without an exit lane (case 1) and that with an exit lane (case 2) are considered. The phase diagram and its variation with L_D (the length of exit lane in case 2 and the length of a special lane-changing section in case 1) is studied. Two phase regions are found in case 1 and a new phase, i.e., the maximum flux phase, is reported in case 2. The density (velocity) distribution near the off ramp and the influence of L_D on the traffic flow are also discussed in both cases.

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I. INTRODUCTION

Recently, traffic and pedestrian flows have attracted considerable attention [1–4]. Different kinds of models, such as car-following models, cellular automaton (CA) models, gas kinetic models, and hydrodynamic models [1–7], have been put forward to study the traffic behavior. In the early 1990s, the rapid development of computer capacity allowed the CA model to display its high practical importance. After Nagel and Schreckenberg [8] introduced a CA single-lane traffic model (NS model), various generalization and extensions of this model are proposed [9–12].

As an important factor in real traffic, bottlenecks have attracted the interest of a number of researchers. The bottlenecks include on-ramps, off-ramps, lane closings, uphill gradients, narrow road sections, etc. Among the various types of bottlenecks, the on-ramp has been widely studied [13–20] with the macroscopic, CA, and car-following models. Many interesting phenomena and useful simulation results have been discovered. In Refs. [21–23] off ramps have been discussed based on experimental observations. It is found that saturated off ramps can have pernicious effects on freeway traffic flow and much more attention should be paid to off ramps [22]. In Ref. [24], both the on ramp and the off ramp are studied by using the single-lane NS model under periodic boundary conditions. It is far away from the real traffic on the freeway. In this paper, we will discuss the traffic behaviors on the freeway near an off ramp with a more realistic CA model.

The paper is organized as follows: In Sec. II we introduce the models used in the simulations. In Sec. III, we discuss the simulation results. At last we reach the final conclusion in Sec. IV.

II. MODEL

In real traffic, an exit lane may be present or absent upstream of an off ramp. We will discuss these two cases in this paper. The case of off ramp without an exit lane is denoted as case 1 and the other is denoted as case 2. As shown in Fig. 1, the system is divided into four sections: sections A, B, C, and D. In sections A and C, a two-lane CA model is used; in section B, the single-lane NS model is adopted; in section D, special lane changing rules should be used.

For the sake of completeness, we briefly recall the definition of the NS model. The NS model is a discrete model for traffic flow. The road is divided into cells which can be either empty or occupied by a car with a velocity $v=0,1,\ldots,v_{max}$. At each discrete time step $t\rightarrow t+1$, the system is updated in parallel according to the following rules: (i) acceleration, $v_n\rightarrow\min(v_{max},v_n+1)$; (ii) deceleration, $v_n\rightarrow\min(v_n,d_n)$; (iii) randomization, $v_n\rightarrow\max(v_n-1,0)$ with probability p; (iv) position update, $x_n\rightarrow x_n+v_n$. Here v_{max} is the maximum velocity of the vehicle, x_n and v_n are the position and velocity of vehicle n, $d_n=x_{n-1}-x_n-1$ is the gap of the vehicle n (it is assumed that vehicle $n-1$ precedes vehicle n), p is randomization probability. This model is used in section B.

The above rules control the forward motion of cars. In the case of two-lane traffic one has to introduce an additional set

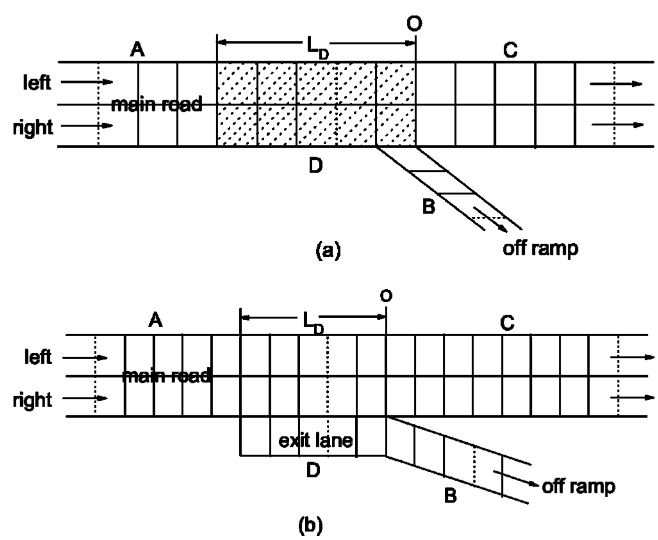

FIG. 1. The schematic illustration of the system: (a) for the case without an exit lane, (b) for the case with an exit lane.

*Corresponding author. Email address: qswu@ustc.edu.cn

of rules that determine the changing cars between lanes. Usually, the update step is divided into two substeps: in the first substep, cars may change lanes in parallel according to lane-changing rules and in the second substep the two lanes are considered as independent single-lane NS models.

For all the through cars on the main road and the exit vehicles in section *A*, we use a symmetric rule set where cars change lanes if the following condition is fulfilled:

$$d_n < \min(v_n + 1, v_{max}) \quad \text{and} \quad d_{n,other} > d_n$$

$$\text{and} \quad d_{n,back} > d_{safe}. \qquad (1)$$

Here $d_{n,other}, d_{n,back}$ denote the number of free cells between the *n*th car and its two neighbor cars on the destination lane at time *t*, respectively; d_{safe} is a safety distance to avoid crash.

In section *D*, the drivers of the exit vehicles usually decelerate because they are near the off ramp. So another maximum velocity v^e_{max} is set for the exit cars. Additionally, the drivers of the exit cars are inclined to run on the right lane for the convenience to exit, so some special lane-changing rules are needed. In *case1*, if condition

$$[d_n = d_{n,other} = 0 \quad \text{or} \quad (d_{n,other} \neq 0 \quad \text{and} \quad d_n - d_{n,other} \leqslant 2)]$$

$$\text{and} \quad d_{n,back} > v_{ob} \qquad (2)$$

is met, the exit car *n* on the left lane will change to the right lane. Here v_{ob} denotes the velocity of the following car on the destination lane. Condition $d_n = d_{n,other} = 0$ means "I cannot move on both lanes in the next time step." Condition $d_n - d_{n,other} \leqslant 2$ means the road condition on present lane is not much better than that on the destination lane. If an exit car still cannot change to the right when it arrives at point *O*, it would stop there and wait for a chance (the corresponding position on the right lane is empty) to change. After updating, if the position of the leading exit car on the right lane (x_{lead}) is greater than x_O, the car would enter the off ramp with a position: ($x_{lead} - x_O$). The exit cars are forbidden to change from right lane to the left in section *D*. In *case2*, the lane-changing rules for the exit cars from left to right and from right to the exit lane are the same as Eq. (2), the lane changes from exit lane to right lane and from right to left are forbidden. The through cars are forbidden to change to the exit lane.

We denote the left and the right lane of roads *A* and *C* as AL, AR, CL, CR, respectively. The boundary conditions are adopted as follows. We assume that the first cells on *AL* and *AR* correspond to $x=1$, and the entrance regions of lanes *AL* and *AL* include v_{max} cells, i.e., the cars can enter lanes *AL* and *AR* from the cells $1, 2, \ldots, v_{max}$. In one time step, when the update of the cars on the road is completed, we check the positions of the last cars on lanes *AL* and *AR* and that of the first car on the road *B*, *CL*, and *CR*, which are denoted as $x_{AL_{last}}$, $x_{AR_{last}}$, and $x_{B_{first}}$, $x_{CR_{first}}$, $x_{CL_{first}}$, respectively. If $x_{AL_{last}}(x_{AR_{last}}) > v_{max}$, a car with velocity v_{max} is injected with probability α at the cell min $[x_{AL_{last}}(x_{AR_{last}}) - v_{max}, v_{max}]$. The entering car is set as an exit one with probability p_{off}, which stands for the percentage of the exit vehicles. Near the exit of the road $C(B)$, the leading car is removed if $x_{CL_{first}}(x_{CR_{first}}) > L_C$ $(x_{B_{first}} > L_B)$ $[L_C$ (L_B) denotes the position of the rightmost cell on road $C(B)]$ and the following car becomes the new leading car and it moves without any hindrance.

III. SIMULATION AND DISCUSSION

In this section, the simulation results are presented. In the simulations, section *C* is divided into $(300 v_{max})$ cells, section *D* into L_D cells and sections *A* and *B* into $[300(v_{max} - L_D)]$ cells. Each cell corresponds to 7.5 m and a vehicle has a length of one cell. One time step corresponds to 1 s. The model parameters $v_{max}=5$, $v^e_{max}=3$, $p=0.3$, $d_{safe}=5$ are used. The first 40 000 time steps are discarded to let the transient time die out. The flux is obtained by counting the vehicles that pass a virtual detector in 100 000 time steps.

A. Case 1

As a preliminary work, we investigate the relationship between the flux and the injection probability α. For the special case $p_{off}=0$, the situation reduces to the two-lane NS model in the open boundary conditions, and the flux on road *A* increases linearly with α and becomes constant Q_{c1} when $\alpha > \alpha_{c1}$.

For another special case $p_{off}=1$, when $\alpha < \alpha_{c2}$ the traffics on both roads *A* and *B* are free flow, and the flux increases linearly with α. When $\alpha \geqslant \alpha_{c2}$ the traffic on road *B* is saturated[1] while that on the road *A* becomes congested. For the case, the flux on road *B* remains a constant with respect to α. Obviously it is the capacity of the off ramp, and we denote it as Q_{c2}. It is clear that Q_{c1} is greater than Q_{c2} because section *A* is a two-lane road while section *B* is single lane, which is a bottleneck in the traffic system.

In Fig. 2, the phase diagram in (α, p_{off}) space is plotted. One can see that two regions are categorized. The traffic flow on the road *A* is free in region I, while it is congested in region II. Because of the introduction of exit cars, section *A* of the main road began to be congested at certain injection probability α_{jam}. With the increase of the percentage of the exit cars (p_{off}), α_{jam} becomes smaller and smaller, i.e., the capacity of the main road is decreased gradually.

Then we focus on the influence of L_D on the traffic behavior. We find that at small value of p_{off} $(p_{off} < p_{c1})$, L_D has a negative effect on the capacity of road *A*; at intermediate value of p_{off}, the increase of L_D improved the road capacity; at large p_{off} value, L_D almost has no effect at all. When $p_{off} < p_{c1}$, the number of exit cars is small and they can easily enter the off ramp, consequently the off ramp almost has no effect on the main road; however, the maximum velocity of the exit cars is set as v^e_{max} in section *D*, thus the increase

[1]When p_{off} is greater than a critical value p_{c0}, the flux of road *B* increases at first with the increase of the number of exit cars entering the system (N_{exit}), however, it does not increase any more with the further increase of N_{exit}. The state of road *B* under such situation is defined as saturated. When $p_{off} < p_{c0}$, road *B* will never get saturated.

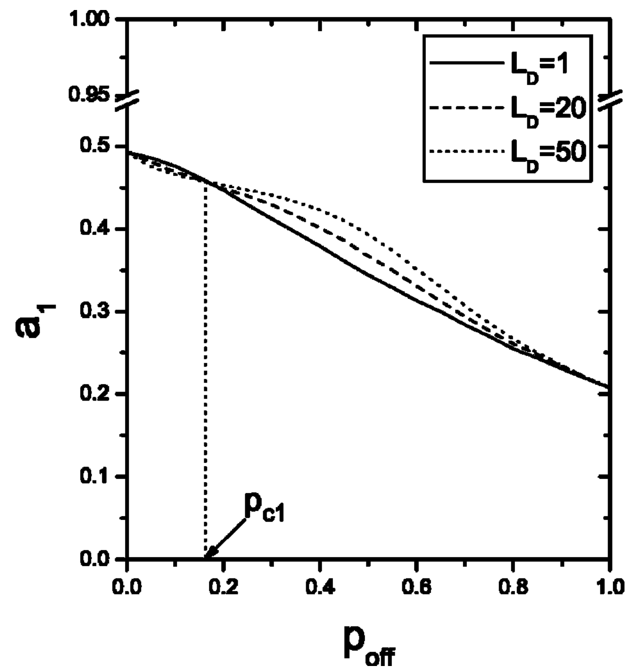

FIG. 2. The phase diagrams in (α, p_{off}) space for the cases of different L_D.

of L_D generates a negative effect. When $p_{off} > p_{c1}$, the exit cars cannot enter the off ramp at will any longer and some of them must stop on the main road to wait for a chance. The special lane-changing rules in section D weaken this situation, as a result, L_D has a positive effect. This can also be seen clearly in Fig. 3: in the case of $P_{off}=0.1$ the capacity of the road A decreases gradually with the increase of L_D, while it increases in the cases of $p_{off}=0.3, 0.4, 0.5$; in the case of $p_{off}=p_{c1}$ the capacity of road A is almost independent of L_D except the several points near $L_D=1$.

From Fig. 3, one can also see that at a given value of L_D, the capacity decreases with the increase of p_{off}. In a recent observation of a freeway diverge [22], it is found that "the freeway discharge flow can change significantly without a

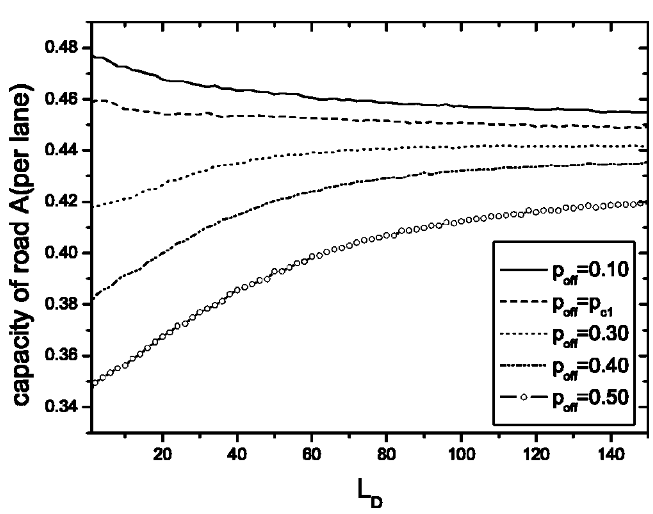

FIG. 3. The relationship between the maximum flux of road A and L_D in the cases of different p_{off}.

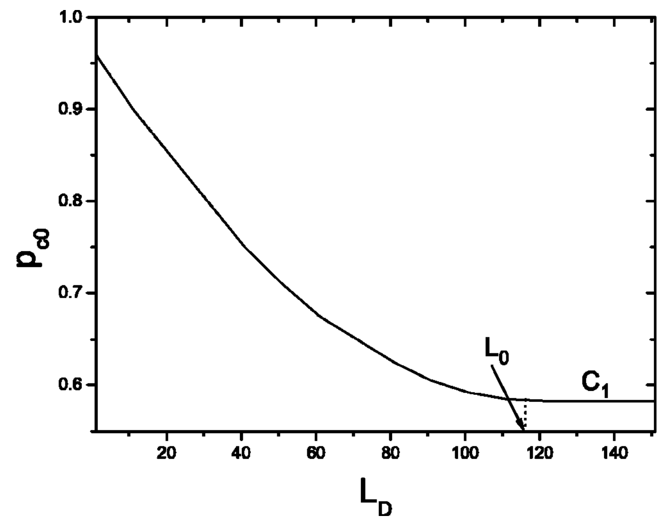

FIG. 4. The variation of p_{c0} against L_D in case 1.

change in the off-ramp flow when the percent of exiting vehicles changes." Our simulations are in accordance with the empirical results.

There exists a critical value p_{c0}, when $p_{off} < p_{c0}$, the off ramp (road B) will never get saturated. It is determined by L_D in *case1*. The relationship between p_{c0} and L_D is shown in Fig. 4. One can find that when $L_D < L_0$, p_{c0} decreases with the increase of L_D, while p_{c0} does not decrease any longer and maintains at a constant when $L_D > L_0$. This suggests that the off ramp can be utilized more efficiently in the case of large L_D than in the case of small L_D.

Next we investigate the density and velocity distributions on the main road near the off ramp. In Figs. 5 and 6, the average density and velocity distributions near the off ramp are demonstrated. $\alpha=0.8$ is taken to ensure the states of sections A and D which are located in region II of the phase diagram. $p_{off}=0.2$ is used in Fig. 5, while $p_{off}=0.5$ is adopted in Fig. 6. Very large p_{off}, which does not exist in real life traffic, is not considered here.

From Fig. 5(a), one can find that the density of left lane ρ_l is almost equal to that of the right lane ρ_r far away upstream of the section D, $\rho_l < \rho_r$ in section D, and $\rho_l > \rho_r$ at the downstream of point O. In addition, two peaks on both lanes are found. As soon as an exit car enters section D, its maximum velocity is reduced to v^e_{max}, i.e., it becomes a slow vehicle,

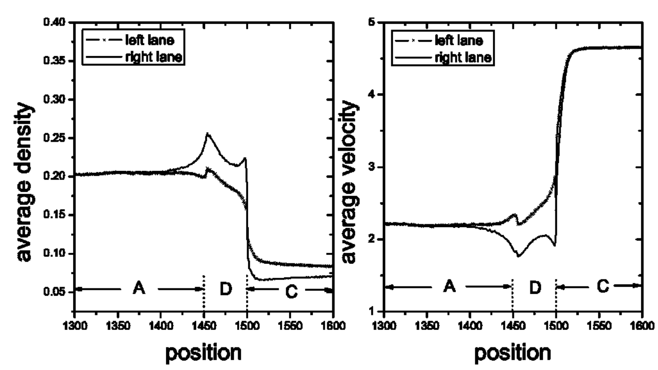

FIG. 5. The density and velocity distributions of the main road near the off ramp in the cases of $L_D=50$ and $p_{off}=0.2$.

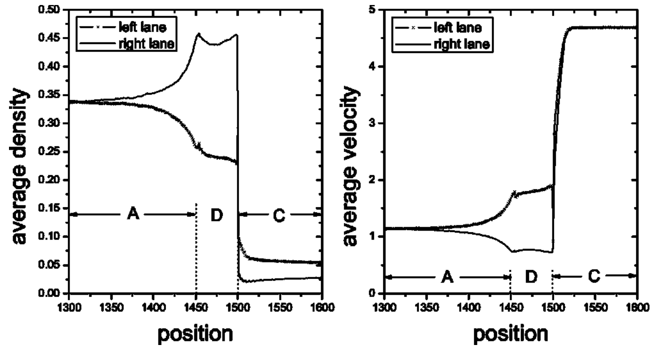

FIG. 6. The density and velocity distributions of the main road near the off ramp in the cases of $L_D=50$ and $p_{off}=0.5$.

and begins to block the following cars. As a result, the first density peak appears near the starting point of section D. If an exit car cannot enter the off ramp when arriving at point O, it will stop there and wait for a chance to change to the right lane or enter the off ramp. Obviously, it becomes a hindrance and thus the second peak appears. Because the exit cars are inclined to run on the right lane in section D, the corresponding ρ_r is much higher than ρ_l. Additionally, most of through cars enter the section C from the left lane because the number of exit cars on the right lane is greater than that on the left lane in section D, so $\rho_l > \rho_r$ downstream of point O.

From Fig. 5(b), one can find the velocity of left lane v_l is almost equal to that of right lane v_r far away upstream of section D and downstream of point O. In section D, v_l is much higher than v_r and two local minima occur. The unequality of the velocities on different lanes in section D is observed in reality, see Ref. [22] where it is reported that queued lanes can move at different speeds.

In the case of $p_{off}=0.5$ (Fig. 6), the corresponding density (velocity) is apparently higher (lower) than that in the case of $p_{off}=0.2$, i.e., the traffic on the sections A and D becomes much more congested. However, no qualitative changes occur.

B. Case 2

In this section, we will investigate the off ramp system with an exit lane. The sketch of the system is shown in Fig. 1(b). Similarly, we discuss the relationship between the flux and the injection probability α under two special cases: $p_{off}=0$ and $p_{off}=1$. For $p_{off}=0$, the situation is the same as that in case 1, while the situation is a little different for $p_{off}=1$: both the capacity (Q_{c3}) and the critical value of α (α_{c3}) are slightly less than Q_{c2} and α_{c2}. This is due to the different rules for the exit car to enter the off ramp.

In Fig. 7, the phase diagrams in the (α, p_{off}) space at different values of L_D are shown. The phase diagram of L_D $=1$ is very similar to that in case 1. However, for the cases of $L_D > 1$, a third region III appears, in which the traffic reaches the maximum flow on the road A [Fig. 7(b)]. With the increase of L_D, region III expands. This is easy to understand. When p_{off} is not so large, the exit cars can easily change to the exit lane and their influence on the through cars is negligible, so the traffic can still reach the maximum flux on road A.

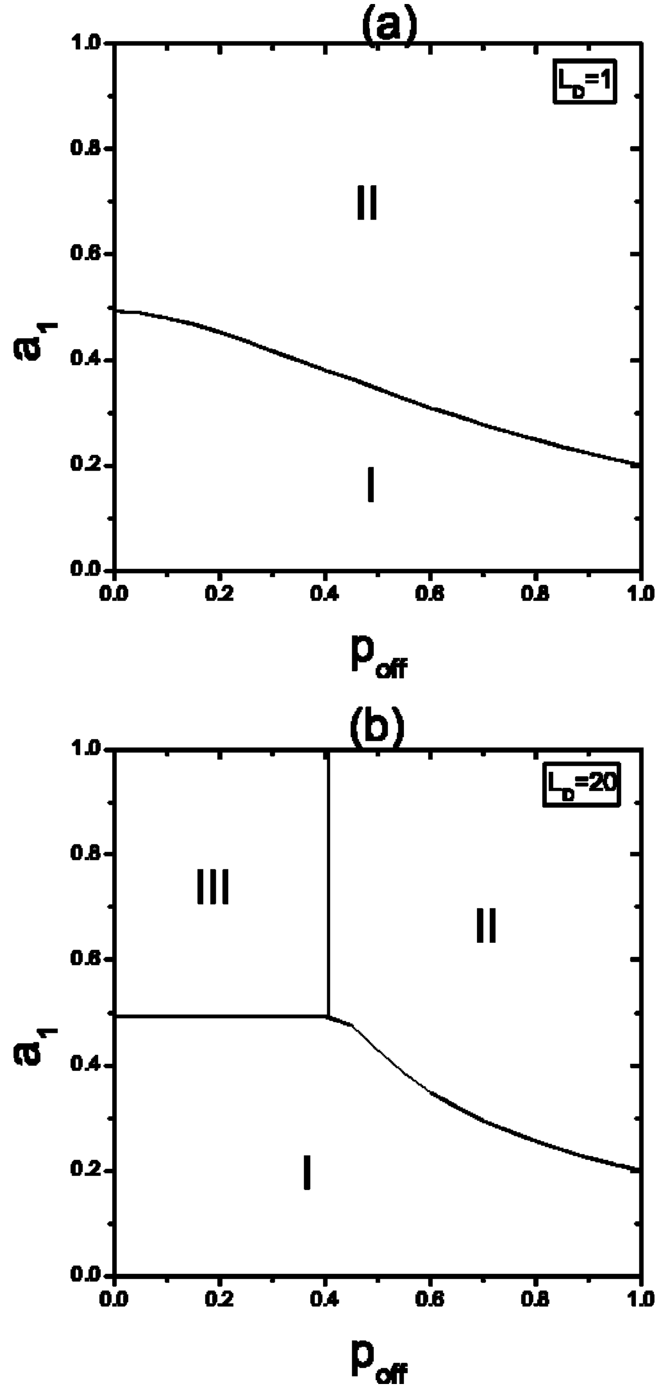

FIG. 7. The phase diagrams in the cases of different L_D.

As for p_{c0} under case 2, it is also determined by L_D. With the increase of L_D, p_{c0} decreases at first and then maintains at a constant C_2 when $L_D > L_1$ (see Fig. 8). Nevertheless, both L_1 and C_2 are smaller than L_0 and C_1 in case 1. This suggests that the road B becomes saturated much more easily in case 2 than in case 1, i.e., the use of ratio of the off ramp is higher in case 2 than in case 1.

We investigate the dependence of the capacity of road A on L_D. The simulation results are demonstrated in Fig. 9. For

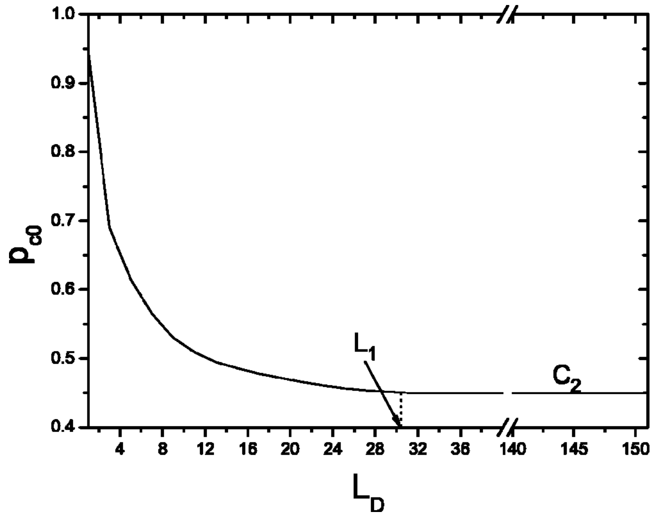

FIG. 8. The variation of p_{c0} against L_D in *case2*.

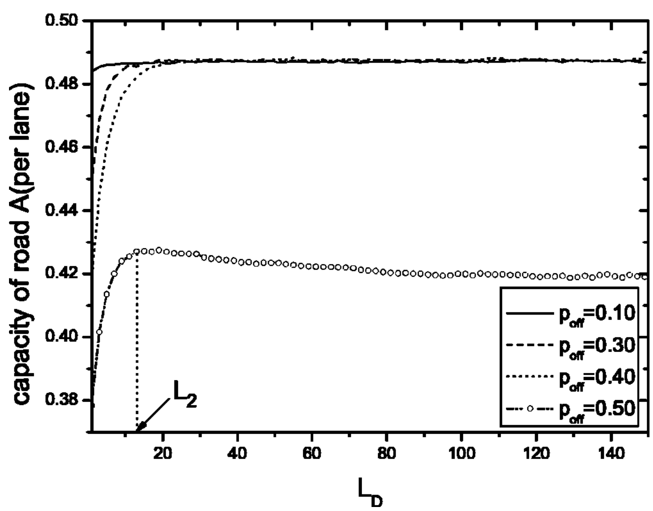

FIG. 9. The relationship between the capacity of road A and L_D in the cases of different p_{off}.

small p_{off}, for example $p_{off}=0.1$, the capacity of the road A is almost not influenced by L_D, and maintains approximately a constant Q_{c1}. With the increase of p_{off}, L_D begins to play an important role in affecting the road capacity. In the cases of $p_{off}=0.2$ and 0.4, the off ramp is not saturated in the whole range of L_D, as a result, the increase of L_D has a positive effect and Q_{c1} can be reached at last. In addition, the larger p_{off} becomes, the longer L_D is needed to reach the capacity of road A.

However, when $p_{off}=0.5$, Q_{c1} cannot be reached any longer. With increase of L_D, the capacity of road A increases at beginning until it reaches a maximum value, and then it decreases with the further increase of L_D. p_{c0} corresponding to L_2 is approximately equal to 0.5 (cf. Fig. 8), i.e., the off ramp is not saturated when $L_D < L_2$, while it is saturated when $L_D > L_2$. One can find that the maximum value in the case of $p_{off}=0.5$ is much less than Q_{c1}. The above simulation results indicate that for a large L_D, the off ramp almost never generates any observable disturbances into the traffic stream before the off ramp gets saturated, while the situation is different after the off ramp becomes saturated.

In a recent study of freeway traffic near an on ramp [23], it is found that whenever the off ramp queues were prevented from spilling over to the exit lane (this implies the off ramp is unsaturated), much higher flows were sustained on the freeway segment and a bottleneck did not arise there. Our simulations are consistent with the experimental observations.

In Figs. 10 and 11, the average density and velocity distributions under case 2 are plotted, where α is also chosen to be 0.8. Far away upstream of section D, $\rho_r = \rho_l (v_r = v_l)$ when $p_{off}=0.2$ or $p_{off}=0.5$. However the density (velocity) is much lower (higher) than that in case 1, i.e., the traffic situation in case 2 is much better than that in case 1 whether the off ramp is saturated or not.

When $p_{off}=0.2$ (Fig. 10), $\rho_r < \rho_l$ at the left part of section D while ρ_r becomes greater than ρ_l at the right part, and $\rho_r > \rho_l$ at the downstream of point O, which are different from the distributions in case 1. It is obviously due to the

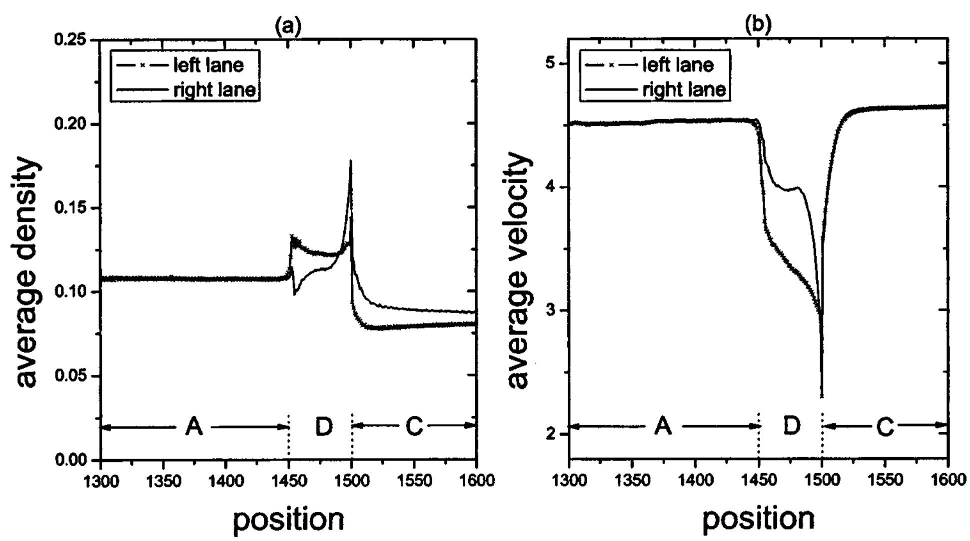

FIG. 10. The density and velocity distributions of the main road near the off ramp in the cases of $L_D=50$ and $p_{off}=0.2$.

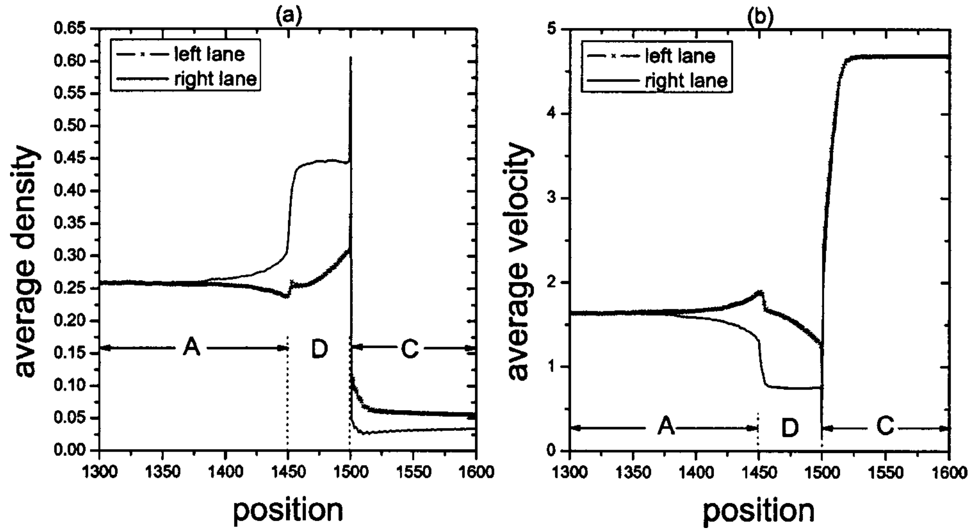

FIG. 11. The density and velocity distributions of the main road near the off ramp in the cases of $L_D = 50$ and $p_{off} = 0.5$.

special lane-changing rules from main road to exit lane. When $p_{off} = 0.5$ (Fig. 11), the off ramp becomes saturated and the exit lane is also congested, so the lane changing from main road to exit lane becomes difficult. As a result the distributions in section D and downstream of point O are qualitatively similar to those in *case1* (cf. Fig. 6).

IV. CONCLUSION

In this paper, we investigated the off ramp system on highway using the cellular automaton traffic model. Two situations, without/with an exit lane near the off ramp, are considered. Phase diagram in (α, p_{off}) space, the influence of L_D on the traffic flow, and the density (velocity) distribution near the off ramp are discussed in detail in both cases (case 1 and case 2).

For case 1. (i) The phase diagram is classified into two regions; (ii) at small value of p_{off}, the increase of L_D has a negative effect on the capacity of the system, while it influences the capacity positively at intermediate value of p_{off}; at large value the influence is negligible; (iii) In section D, ρ_r (v_r) is greater (less) than ρ_l (v_l), and two peaks (local minimum) are found in the density (velocity) distribution.

For case2. For this case, we have the following.

(i) A new phase region (maximum flow region) is reported, and it expands with the increase of L_D.

(ii) At small value of p_{off}, the capacity of the system is almost not infected by L_D and maintains approximately at Q_{c1} which is the maximum flux of the system; at intermediate value of p_{off} (the off ramp is not saturated yet), L_D has a positive effect, the larger p_{off} is, the longer L_D is needed to reach Q_{c1}; at large value of p_{off} (off ramp becomes saturated when $L_D > L_2$), Q_{c1} cannot be reached any longer, with the increase of L_D, the capacity increases to a maximum value and then decreases gradually.

(iii) The density (velocity) distribution is different from that in case 1 for small p_{off}. $\rho_r < \rho_l (v_r > v_l)$ at the left part of section D, while the situation is opposite at the right part. $\rho_r > \rho_l$ downstream of the off ramp.

Our simulation results suggest that the traffic situations in case 2 are better than those in case 1, i.e., an exit lane is very useful at the off ramp. With respect to the L_D, it is not the longer the better, a proper value should be designed to get an optimal capacity.

Finally, we would like to mention that in our work, the symmetric lane-changing rule is applied to all the through cars on the main road and the exit vehicles in section A for simplification reasons. However, recent investigations [25] have shown that it is difficult to define realistic lane-changing rules and that they may easily produce artifacts. Therefore, more efforts will be done in our future work to study the traffic behaviors under different lane-changing rules and compare the results with empirical data.

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